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Superstring interactions in a pp-wave background II

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Abstract: In type-IIB light-cone superstring field theory, the cubic interaction has two pieces: a delta-functional overlap and an operator inserted at the interaction point. In this paper we extend our earlier work [8] by computing the matrix elements of this operator in the oscillator basis of pp-wave string theory for all $\mu p^+ \alpha'$. By evaluating these matrix elements for large $\mu p^+ \alpha'$, we check a recent conjecture relating matrix elements of the light-cone string field theory hamiltonian (with prefactor) to certain three-point functions of BMN operators in the gauge theory. We also make several predictions for gauge theory.

Keywords: String Field Theory, Penrose limit and pp-wave background.

Contents

1. Introduction

The proposal of [1] equating string theory in a pp-wave background [2, 3] to a certain sector of $\mathcal{N} = 4$ SU(N) Yang-Mills theory containing operators of large R-charge J involves a limit in which the two parameters

$$\lambda' = \frac{g_{\rm YM}^2 N}{J^2} = \frac{1}{(\mu p^+ \alpha')^2}\,, \qquad g_2 = \frac{J^2}{N} = 4\pi g_s (\mu p^+ \alpha')^2 \tag{1.1}$$

are held fixed while N and the 't Hooft coupling $\lambda = g_{\rm YM}^2 N$ are taken to infinity (see also [4]). It has recently been proposed [5, 6, 7] to extend this duality to include string interactions. The existence of two expansion parameters (1.1) opens up the intriguing possibility that there is a regime in which both the string and gauge theories are effectively perturbative.

The authors of [7] postulated a correspondence between Yang-Mills perturbation theory and string perturbation theory of the form

$$\langle i|H|j\rangle|k\rangle = \mu(\Delta_i - \Delta_j - \Delta_k) C_{ijk}\,, \tag{1.2}$$

to leading order in λ', and for a particular class of states. Here i, j, k label three free string states in the pp-wave background and the left-hand side is the matrix element of the interacting light-cone string field theory hamiltonian which connects the two-string state $|j\rangle|k\rangle$ to the single string state $\langle i|$. On the right-hand side, C_{ijk} is the coefficient of the three-point function for the operators corresponding to the three string states and Δ_i is the conformal dimension of operator i (equivalently, the mass of string state i). The conjecture (1.2) seemed to pass a nontrivial test in a check of the one-loop mass renormalization of the pp-wave string spectrum [7]. Although the leading (in g_2) term in the light-cone string field theory hamiltonian was determined formally in [8] for all λ', the precise matrix elements investigated in [7] had not been checked explicitly in string theory.

Type-IIB light-cone superstring field theory for the pp-wave was constructed in [8]. The cubic interaction has two parts: a delta-functional overlap which expresses continuity of the string worldsheet, and the 'prefactor', which is an operator required by supersymmetry that is inserted at the point where the string splits. At $\lambda' = 0$ (infinite μ), it is easy to see [7] that the rules for calculating the delta-functional overlap agree with the rules for calculating the three-point functions of BMN operators (see also [9], and [10] for the leading correction to this result). Therefore the nontrivial part of the proposal (1.2) is that the effect of this prefactor (for large μ) is simply to multiply the delta-functional overlap by the factor $\Delta_i - \Delta_j - \Delta_k$.[1] In this paper we compute the matrix elements of this prefactor in the oscillator basis of pp-wave string theory for all λ'. The leading λ' behavior of the matrix elements we calculate is not consistent with the proposal (1.2). This discrepancy in no way invalidates the BMN correspondence, since matrix elements of the hamiltonian are not invariant quantities, but depend on a choice of basis. That is, the *eigenvalues* of the interacting hamiltonian, as computed on both sides of the duality, should agree, but there is no reason to expect the *matrix elements* of the left-hand side of (1.2), computed in the natural string field theory basis (of single- and multi-string states), to agree with the matrix elements of the right-hand side of (1.2), computed in the natural field theory basis (of single- and multi-trace operators).

The most significant difficulty in this enterprise is the fact that the limits $\mu \to \infty$ (high curvatures in spacetime) and $n \to \infty$ (short distances on the worldsheet) do not commute. Specifically, the prefactor arises entirely from a short-distance effect on the string worldsheet and must therefore be determined first at finite μ, with the $\mu \to \infty$ limit taken at the end. This technical difficulty raises an important conceptual question as well, since it would seem that the prefactor cannot possibly be seen in perturbative gauge theory expanded around $\mu = \infty$. Clearly, a deeper understanding of how these two theories are related is very desirable.

The plan of the paper is the following. In section 2 we review the relevant gauge theory results and give a brief overview of the three-string vertex. In section 3 we determine the prefactor in the oscillator basis for all μ and present a simple formula valid for large μ. In section 4 we calculate the leading $\mathcal{O}(\lambda')$ term in the matrix elements and compare with the proposal (1.2).

[1] The fact that the effective strength of string interactions in the pp-wave background scales as the energy of the string states involved has been argued in [11].

Related work includes the matrix string theory approach to string interactions in the pp-wave background [11], open string interactions between D-branes in a pp-wave [12], and matrix models for M-theory on the maximally supersymmetric pp-wave [13].

After this work was completed, the paper [14] appeared which presents a string bit formalism for interacting strings in the pp-wave background, expected to be valid to all orders in g_2 and leading order in λ'. This complements the work of our paper, since we work at leading order in g_2 but for all λ'.

2. Overview

In this section we summarize the relevant results of [1, 7] and [8] and write the matrix elements which will be calculated from string theory in section 4.

2.1 Field theory

Following [1, 7] we consider the chiral operators

$$\begin{aligned} O^J &= \frac{1}{\sqrt{JN^J}} \operatorname{Tr}\left(Z^J\right), \\ O^J_\phi &= \frac{1}{\sqrt{N^{J+1}}} \operatorname{Tr}\left(\phi Z^J\right), \\ O^J_\psi &= \frac{1}{\sqrt{N^{J+1}}} \operatorname{Tr}\left(\psi Z^J\right), \end{aligned} \tag{2.1}$$

as well as the operator

$$O^J_n = \frac{1}{\sqrt{JN^{J+2}}} \sum_{k=0}^{J} e^{2\pi i n k/J} \operatorname{Tr}\left(\phi Z^k \psi Z^{J-l}\right), \tag{2.2}$$

which is chiral for $n = 0$ and "almost chiral," in a controllable sense, for $n/J \ll 1$. Here ϕ, ψ and Z are three orthogonal complex linear combinations of the six real scalar fields of $\mathcal{N} = 4$ Yang-Mills theory. Without loss of generality we can take

$$\phi = \frac{X^1 + iX^2}{\sqrt{2}}, \qquad \psi = \frac{X^3 + iX^4}{\sqrt{2}}, \qquad Z = \frac{X^5 + iX^6}{\sqrt{2}}. \tag{2.3}$$

Free string theory in the pp-wave background is exactly solvable [2]. The anomalous dimensions of the operators (2.1) and (2.2) were computed in [1], leading to the following identification between these BMN operators and string states:[2]

$$\begin{aligned} O^J &\leftrightarrow |0; p^+\rangle, \\ O^J_\phi &\leftrightarrow \alpha_0^{\phi\dagger}|0; p^+\rangle, \\ O^J_\psi &\leftrightarrow \alpha_0^{\psi\dagger}|0; p^+\rangle, \\ O^J_n &\leftrightarrow \alpha_n^{\phi\dagger}\alpha_{-n}^{\psi\dagger}|0; p^+\rangle, \end{aligned} \tag{2.4}$$

[2] We use α to denote the oscillators which were called a in [1] and [7]. A different basis, related for $n \neq 0$ by $\alpha_n = \frac{1}{\sqrt{2}}(a_{|n|} - i \operatorname{sign}(n) a_{-|n|})$ and $\alpha_0 = a_0$, appears in [8] and will be used later in this paper.

where $\alpha^{\phi} = \frac{1}{\sqrt{2}}(\alpha^1 - i\alpha^2)$, $\alpha^{\psi} = \frac{1}{\sqrt{2}}(\alpha^3 - i\alpha^4)$, and J and p^+ are related by

$$J = \sqrt{\lambda}\mu p^+ \alpha' \,. \tag{2.5}$$

The dictionary (2.4) is precise only at lowest order in λ' and g_2. At higher orders some of the operators receive wavefunction renormalizations and mix with each other (as well as with multi-trace operators) and the set of BMN operators must be diagonalized in order to preserve the correspondence with single string states. We will use the notation $|O^J\rangle$, $|O^J_\phi\rangle$, etc. to refer to the states in (2.4), without any corrections.

Using the proposal (1.2), the matrix elements

$$\begin{aligned} \langle O^J_n|H|O^{J_1}_m\rangle|O^{J_2}\rangle &= \frac{4g_s\mu}{\pi}(1-y)\frac{(ny+m)}{(ny-m)}\sin^2(\pi n y)\,, \\ \langle O^J_n|H|O^{J_1}_\phi\rangle|O^{J_2}_\psi\rangle &= \frac{4g_s\mu}{\pi}\sqrt{y(1-y)}\sin^2(\pi n y)\,, \end{aligned} \tag{2.6}$$

were obtained in [7] from field theory calculations[3]. Here $J = J_1 + J_2$ and $y = J_1/J$. The goal of this paper is to check whether these matrix elements can be obtained from light-cone string field theory using the proposal (1.2).

2.2 String theory

The cubic term in the light-cone string field theory hamiltonian in the pp-wave background was determined in [8]. As explained in the introduction, it consists of two pieces. The first is the delta-functional overlap, which can be expressed as a state $|V\rangle$ in the three-string Hilbert space as

$$|V\rangle = \exp\left[\frac{1}{2}\sum_{r,s=1}^{3}\sum_{m,n=-\infty}^{\infty} a^{I\dagger}_{m(r)}\overline{N}^{(rs)}_{mn}a^{J\dagger}_{n(s)}\delta_{IJ}\right]|0\rangle\,. \tag{2.7}$$

Here $r,s \in \{1,2,3\}$ label the three strings and $I,J \in \{1,\ldots,8\}$ label the transverse directions. An expression for the Neumann matrix elements $\overline{N}^{(rs)}_{mn}$ was given in [8]. We have omitted the fermionic terms which will make a brief but significant appearance below. The second piece is the prefactor which we represent as an operator $\widehat{H}$ acting on the delta-functional[4] $|H\rangle = 2\pi g_s\mu\widehat{H}|V\rangle$. It is given by

$$\widehat{H} = K^I\widetilde{K}^I v_{IJ}(\Lambda)\,, \tag{2.8}$$

where the operators K, $\widetilde{K}$ and Λ are linear in string creation operators (Λ being fermionic), and v^{IJ} is the tensor

$$\begin{aligned} v^{IJ} = {}& \delta^{IJ} - \frac{i}{\alpha}\gamma^{IJ}_{ab}\Lambda^a\Lambda^b + \frac{1}{6\alpha^2}\gamma^{IK}_{ab}\gamma^{JK}_{cd}\Lambda^a\Lambda^b\Lambda^c\Lambda^d - \frac{4i}{6!\alpha^3}\gamma^{IJ}_{ab}\epsilon_{abcdefgh}\Lambda^c\Lambda^d\Lambda^e\Lambda^f\Lambda^g\Lambda^h + \\ & + \frac{16}{8!\alpha^4}\delta^{IJ}\epsilon_{abcdefgh}\Lambda^a\Lambda^b\Lambda^c\Lambda^d\Lambda^e\Lambda^f\Lambda^g\Lambda^h\,. \end{aligned} \tag{2.9}$$

[3] We use the relativistic normalization $\langle i|j\rangle = p^+_i\delta(p^+_i - p^+_j)$, and the matrix elements (2.6) both contain a factor of $p^+\delta(p^+ - p^+_1 - p^+_2)$ which we suppress.

[4] The overall normalization was not determined in [8]. In [7] it was argued that the effective string coupling for the class of states under consideration is $g_s\mu$, and we employ this result here.

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Here Λ is a positive chirality SO(8) spinor, $a, b, \ldots$ are spinor indices, and we use the notation $\alpha_{(r)} = \alpha' p^+_{(r)}$, with $\alpha \equiv \alpha_{(1)}\alpha_{(2)}\alpha_{(3)}$. Formal expressions for the operators K, $\widetilde{K}$ and Λ were presented in [8]. In order to compare with (2.6) we will need to calculate the matrix elements of K and $\widetilde{K}$ in the number basis of pp-wave string theory.

Since none of the states of interest in (2.6) have fermionic excitations (i.e., they are all primary string states[5]) we will not need to know the matrix elements of Λ, and indeed we can set all fermionic non-zero modes in Λ to zero, so that $\Lambda = \alpha_{(1)}\lambda_{(2)} - \alpha_{(2)}\lambda_{(1)}$ in terms of the fermionic zero modes $\lambda_{(r)}$. The supermultiplet structure in the pp-wave background was studied in detail in [3], and it was shown that primaries live in the λ_{R}^4 component of the type-IIB superfield. (Recall that the SO(8) transverse symmetry of the pp-wave background is broken to SO(4) $\times$ SO(4) by a mass term for the fermions proportional to $\Pi = \Gamma^1\Gamma^2\Gamma^3\Gamma^4$, and we are using the notation $\lambda_{\mathrm{R}} = \frac{1}{2}(1+\Pi)\lambda$.) A simple counting of fermionic zero modes shows that only the quartic term in v^{IJ} has the right number of fermions to contribute to the matrix element of primary states, so for our purposes we need only

$$v^{IJ} = \frac{1}{6\alpha^2}\gamma^{IK}_{ab}\gamma^{JK}_{cd}\Lambda^a\Lambda^b\Lambda^c\Lambda^d \,. \tag{2.10}$$

It was pointed out in[6] that the tensor structure of this term has a remarkable property when the spinor indices a, b, c, d are restricted to the same SO(4) subgroup (which will be the case for primaries):

$$\gamma^{iK}_{[ab}\gamma^{jK}_{cd]} = \delta^{ij}\epsilon_{abcd}\,, \qquad \gamma^{i'K}_{[ab}\gamma^{j'K}_{cd]} = -\delta^{i'j'}\epsilon_{abcd}\,, \qquad \gamma^{iK}_{[ab}\gamma^{j'K}_{cd]} = 0\,, \tag{2.11}$$

where $i, j \in \{1,2,3,4\}$ and $i', j' \in \{5,6,7,8\}$. Recalling that string oscillations in the directions 1,2,3,4 correspond in the Yang-Mills theory to the insertion of ϕ and ψ impurities into a string of Z's, while string oscillations in the directions 5,6,7,8 correspond to the insertion of $D_\mu Z$ impurities, the formula (2.11) gives a simple prediction: the matrix elements analogous to (2.6) should change only by a sign when the ϕ and ψ are replaced by two $D_\mu Z$'s, while they should vanish for matrix elements with one ϕ or ψ and one $D_\mu Z$. This prediction holds not just for the supergravity modes considered in [15], but indeed for all primary string states. It would be interesting to see how the $\mathbb{Z}_2$ symmetry between the two SO(4)'s arises in the field theory, where no symmetry between the two SO(4)'s seems manifest (see [16]).

3. The prefactor in the oscillator basis

The bosonic operators K, $\widetilde{K}$ which appear in the prefactor are

$$K = K_+ - K_-\,, \qquad \widetilde{K} = K_+ + K_-\,, \tag{3.1}$$

[5]We emphasize that the operator (2.2) is not a chiral primary in the *field* theory, but the state it corresponds to in the *string* theory is primary, i.e., the lightest member of a supermultiplet. Whenever we say 'primary' we mean a string state with no fermionic excitations, not (necessarily) a supergravity state, which would correspond to a chiral primary operator in the field theory.

[6]Also L. Motl, private communication, and [15].

where $K_\pm$ may be defined by their action on $|V\rangle$ by the formulas

$$
\begin{aligned}
K_+|V\rangle &= -2\pi\sqrt{-\alpha} \lim_{\sigma\to\pi\alpha_{(1)}} (\pi\alpha_{(1)} - \sigma)^{1/2}(P_{(1)}(\sigma) + P_{(1)}(-\sigma))|V\rangle, K_-|V\rangle \\
&= -2\pi\sqrt{-\alpha} \lim_{\sigma\to\pi\alpha_{(1)}} (\pi\alpha_{(1)} - \sigma)^{1/2}\frac{1}{4\pi}(\partial_\sigma X_{(1)}(\sigma) + \partial_\sigma X_{(1)}(-\sigma))|V\rangle
\end{aligned} \tag{3.2}
$$

(see [8] for details). Note that we suppress the transverse SO(8) index I throughout this section, since it plays no role. Using the expansions of P and X in terms of string modes [8], it is easy to see that K_+ will be linear in oscillators $a_m^\dagger$ with non-negative indices m, and K_- will be linear in oscillators with negative indices, i.e., they will be of the form

$$
K_+ = \sum_{r=1}^{3} F^0_{(r)} a^\dagger_{0(r)} + \sum_{r=1}^{3}\sum_{m=1}^{\infty} F^+_{m(r)} a^\dagger_{m(r)}\,, \qquad K_- = \sum_{m=1}^{\infty} F^-_{m(r)} a^\dagger_{-m(r)}\,, \tag{3.3}
$$

where the components F are to be determined. We use the notation $F_{n(r)}$ to collect F^-, F^0 and F^+ into a single vector with index n from $-\infty$ to $+\infty$, so that

$$
K = \sum_{r=1}^{3}\sum_{m=-\infty}^{\infty} F_{m(r)} a^\dagger_{m(r)}\,. \tag{3.4}
$$

Obtaining the matrix elements $F_{n(r)}$ from the definition (3.2) is tricky, so we will use an alternate approach. The operator K may also be determined by enforcing invariance under the pp-wave superalgebra (see [8] for details). In particular, it must satisfy

$$
\left[\textstyle\sum_{r=1}^{3} P_{(r)}(\sigma), K\right] = \left[\textstyle\sum_{r=1}^{3} e_{(r)} X_{(r)}(\sigma), K\right] = 0\,, \tag{3.5}
$$

where $e_{(r)} = \operatorname{sign}(p^+_{(r)})$, which we take to be $+1$ for $r = 1, 2$ (incoming strings) and -1 for $r = 3$ (outgoing string). Substituting (3.4) into (3.5) and using the mode expansions from [8], we find the equations

$$
\sum_{r=1}^{3} X^{(r)} C_{(r)}^{1/2} F_{(r)} = \sum_{r=1}^{3} \alpha_{(r)} X^{(r)} C_{(r)}^{-1/2} F_{(r)} = 0\,. \tag{3.6}
$$

The matrices $X^{(r)}$ and $C_{(r)}$, as well as all other matrices which appear later in this section, are cataloged in appendix A. From the form of the overlap matrix $X^{(r)}$ it is clear that the negative and non-negative components of (3.6) decouple. Thus there are two linearly independent solutions to (3.5), which is good since we need precisely the two linear combinations (3.1) of the negative and non-negative modes.

The zero-component of the vector equation (3.6) is immediately solved to give

$$
F^0_{(1)} = -\sqrt{\mu\alpha_{(1)}}\,\alpha_{(2)}\,, \qquad F^0_{(2)} = \sqrt{\mu\alpha_{(2)}}\,\alpha_{(1)}\,, \qquad F^0_{(3)} = 0\,. \tag{3.7}
$$

The overall normalization, which is not fixed by (3.5), is determined by comparing to the prefactor for the three superparticle vertex obtained in [8]. Now we proceed to solve for the positive and negative index parts of F.

3.1 Positive indices

For positive indices we obtain upon substituting (3.7) the equations

$$\begin{aligned}\sum_{r=1}^{3} A^{(r)} C^{-1/2} C_{(r)}^{1/2} F_{(r)}^{+} &= \frac{1}{\sqrt{2}} \mu \alpha B \,, \\ \sum_{r=1}^{3} \alpha_{(r)} A^{(r)} C^{-1/2} C_{(r)}^{-1/2} F_{(r)}^{+} &= \frac{1}{\sqrt{2}} \alpha B \,.\end{aligned} \tag{3.8}$$

By taking an appropriate linear combination of (3.8) to eliminate the right-hand side, we find

$$\sum_{r=1}^{3} A^{(r)} C^{1/2} C_{(r)}^{-1/2} U_{(r)} F_{(r)}^{+} = 0 \,. \tag{3.9}$$

Using the first identity in (A.4) to solve (3.9) gives

$$F_{(r)}^{+} = \frac{1}{\alpha_{(r)}} C_{(r)}^{1/2} C^{1/2} U_{(r)}^{-1} A^{(r)\mathrm{T}} V \,, \tag{3.10}$$

where V is an arbitrary vector. Plugging (3.10) into the second equation determines V and hence the complete solution

$$F_{(r)}^{+} = \frac{1}{\sqrt{2}} \frac{\alpha}{\alpha_{(r)}} C_{(r)}^{1/2} C^{1/2} U_{(r)}^{-1} A^{(r)\mathrm{T}} \Upsilon^{-1} B \,, \qquad \Upsilon \equiv \sum_{r=1}^{3} A^{(r)} U_{(r)}^{-1} A^{(r)\mathrm{T}} \,. \tag{3.11}$$

It is easy to see that one recovers the flat space expression of [17] upon setting μ to zero (modulo one factor of $C^{1/2}$ which arises from a different normalization of the string oscillators).

3.2 Negative indices

For negative indices the corresponding equations are

$$\begin{aligned}\sum_{r=1}^{3} \frac{1}{\alpha_{(r)}} A^{(r)} C^{1/2} C_{(r)}^{1/2} F_{(r)}^{-} &= 0 \,, \\ \sum_{r=1}^{3} A^{(r)} C^{1/2} C_{(r)}^{-1/2} F_{(r)}^{-} &= 0 \,.\end{aligned} \tag{3.12}$$

These equations are more subtle than the ones from the previous subsection. This is because (as in flat space [17]), it turns out that the solution for F^- is such that the expressions (3.12) actually diverge when F^- is substituted. However, when one goes back to the function of σ responsible for the divergence, one finds that it is of the form $\delta(\sigma - \pi\alpha_{(1)}) - \delta(\sigma + \pi\alpha_{(1)})$, which we are allowed to ignore, since those two points are identified (see [17] for details).

Instead of solving (3.12) directly, we can bypass this subtlety by appealing to a trick. In flat space, the trick one can appeal to is a very simple relation between positive-index and negative-index Neumann matrix elements. In flat space that relation is a consequence of conformal invariance on the worldsheet, which we do not have in light-cone gauge for pp-wave string theory. Nevertheless we will be able to obtain a fairly simple relation which suffices.

Recall the definition (3.2) of $K_\pm$. Using the expansion of $P(\sigma)$ and $X(\sigma)$ in terms of modes, and using (2.7), one finds that

$$F_{n(r)} \sim \lim_{\sigma\to\pi\alpha_{(1)}} \left(\pi\alpha_{(1)}-\sigma\right)^{1/2} \sum_{p=-\infty}^{\infty} s(p)\sqrt{|p|}\cos\frac{p\sigma}{\alpha_{(1)}}\overline{N}^{(1r)}_{pn}\,, \tag{3.13}$$

where $s(p)=1$ for $p\geq 0$ and $s(p)=-i$ for $p<0$. The $\epsilon^{-1/2}$ singularity near $\sigma=\pi\alpha_{(1)}-\epsilon$ comes entirely from the large p behavior of this sum. Therefore, we can determine a relation between F^+ and F^- by determining a relation between Neumann matrix elements for positive and negative indices. For example, the relation

$$\left[\overline{N}^{(s3)}\right]_{-p,-n} = -\left[U_{(s)}\overline{N}^{(s3)}U_{(3)}\right]_{pn}\,, \qquad s\in\{1,2\} \tag{3.14}$$

is proven in appendix B, and other necessary relations are easily proven for other components. Note that the factor $U_{(s)}$ on the left goes to 1 for large p, so it does not affect the large p behavior. We obtain from (3.14) and the analogous relations for other r,s the remarkably simple relation

$$F^-_{(r)} = iU_{(r)}F^+_{(r)}\,. \tag{3.15}$$

3.3 Large μ expansion

The expressions (3.7), (3.11) and (3.15) determine the matrix elements of the prefactor for all μ. They are more explicit than the formulas (3.2) which were presented in [8] in the continuum basis, but we are still one step away from being able to precisely compare our matrix elements to (2.6): we need to expand these expressions to leading order for large μ. Unfortunately the formula (3.11) is not well-suited for this purpose. One can show that

$$\Upsilon^{-1}B \sim CB + \mathcal{O}\left(\mu^{-1}\right) \tag{3.16}$$

up to an overall (μ-independent) coefficient. The leading term is annihilated by $A^{(r)\mathrm{T}}$ for $r\in\{1,2\}$ and hence gives no contribution to (3.11), while the subleading term has the property that multiplying on the left by $A^{(r)\mathrm{T}}$ gives a divergent sum. This problem alerts us to the fact that for $r\in\{1,2\}$ the large μ expansion of (3.16) does not commute with multiplication by $A^{(r)\mathrm{T}}$, so one has to first calculate the entire matrix $A^{(r)\mathrm{T}}\Upsilon^{-1}B$ and then take the limit. Only for the $r=3$ component is it sensible and consistent to use (3.16), which leads to

$$F^{(3)}_+ \sim -\mu^{-1/2}\frac{1}{2\sqrt{2}}\frac{\alpha}{|\alpha_{(3)}|^{3/2}}C^{5/2}B\,. \tag{3.17}$$

To find the leading behavior of the $r\in\{1,2\}$ components it is easier to return to the original equations (3.8), which for large μ degenerate into the single equation

$$\sum_{r=1}^{3}\sqrt{\mu\alpha_{(r)}}A^{(r)}C^{-1/2}F^+_{(r)} = \frac{1}{\sqrt{2}}\mu\alpha B\,, \tag{3.18}$$

whose solution[7] is

$$F^+_{(r)} = -\sqrt{\frac{\mu}{2}}\,\frac{\alpha\alpha_{(3)}}{\alpha_{(r)}^{3/2}}C^{3/2}A^{(r)\mathrm{T}}C^{-1}B\,, \qquad r\in\{1,2\}\,. \tag{3.19}$$

Using (A.2) and (A.12) allows us to write explicit expressions for the matrix elements to leading order for large μ,

$$\begin{aligned} F^+_{m(1)} &= \sqrt{2\mu\alpha_{(1)}}\alpha_{(2)}(-1)^{m+1}\,,\\ F^+_{m(2)} &= \sqrt{2\mu\alpha_{(2)}}\alpha_{(1)}\,,\\ F^+_{m(3)} &\sim \frac{1}{\pi\sqrt{2\mu|\alpha_{(3)}|}}(-1)^m m\sin(m\pi\beta)\,. \end{aligned} \tag{3.20}$$

Finally the relation (3.15) tells us that to leading order we have

$$F^-_{(r)n} = i\frac{n}{2\mu\alpha_{(r)}}F^+_{(r)n}\,, \quad r\in\{1,2\}\,, \qquad F^-_{(3)n} = -i\frac{2\mu\alpha_{(3)}}{n}F^+_{(3)n}\,. \tag{3.21}$$

4. String theory matrix elements

In this section we use the results of the previous section to calculate the matrix elements (2.6) from string theory for large μ. In [8] the cubic interaction in the light-cone string field theory hamiltonian was determined up to an overall function $f(\mu p^+\alpha')$. A careful analysis of supergravity scattering in the pp-wave background [15] suggests that there the corresponding function is a numerical constant, although it is not clear that this continues to be true in the full string theory. We will use the symbol $\sim$ in this section to indicate this assumption about the overall normalization.

Since the polarization of all excitations lies in the first SO(4), the analysis at the end of section 2 shows that for these matrix elements, the prefactor may be taken to be

$$\widehat{H} = \delta^{ij}\left(K^i_+ + K^i_-\right)\left(K^j_+ - K^j_-\right), \qquad i,j\in\{1,\ldots,4\}\,. \tag{4.1}$$

Using this and the expressions (2.7) and (3.3), it is simple to show that the desired matrix elements are

$$\begin{aligned} \langle O^J_n|H|O^{J_1}_m\rangle|O^{J_2}\rangle &\sim 2\pi g_s\mu^2\left(F^+_{m(1)}F^+_{n(3)} + F^-_{m(1)}F^-_{n(3)}\right)\left(\overline{N}^{(13)}_{m,n} - \overline{N}^{(13)}_{-m,-n}\right),\\ \langle O^J_n|H|O^{J_1}_\phi\rangle|O^{J_2}_\psi\rangle &\sim 2\pi g_s\mu^2 F_{n(3)}\left(F_{0(1)}\overline{N}^{(23)}_{0,n} + F_{0(2)}\overline{N}^{(13)}_{0,n}\right) \end{aligned} \tag{4.2}$$

(written in the a oscillator basis—see footnote 2). We stress that the results (4.2) are correct for all μ. In order to compare with (2.6) for large μ, we use the fact that at $\mu=\infty$ the only nonzero Neumann matrices are

$$\overline{N}^{(r3)} = -\sqrt{\frac{\alpha_{(r)}}{|\alpha_{(3)}|}}X^{(r)\mathrm{T}}\,, \qquad r\in\{1,2\} \tag{4.3}$$

[7] One might be worried that a single equation is not sufficient to determine two quantities $F^{(1)}$ and $F^{(2)}$. However, $A^{(1)}$ and $A^{(2)}$ have orthogonal images and no kernels, so the equation $A^{(1)}v_{(1)} + A^{(2)}v_{(2)} = 0$ has no nontrivial solutions. Hence the solution to (3.18) is indeed unique.

(and their transposes $\overline{N}^{(3r)}$). Using the expressions from appendix A, as well as the results (3.20) and (3.21), we immediately obtain

$$\begin{aligned} \langle O_n^J | H | O_m^{J_1} \rangle | O^{J_2} \rangle &\sim \frac{4 g_s \mu}{\pi} (1+\beta) \sin(n\pi\beta)^2 , \\ \langle O_n^J | H | O_\phi^{J_1} \rangle | O_\psi^{J_2} \rangle &\sim \frac{4 g_s \mu}{\pi} \sqrt{-\beta(1+\beta)}\, \sin(n\pi\beta)^2 \,. \end{aligned} \tag{4.4}$$

The second matrix element agrees precisely with (2.6) after recalling that $\beta = -y$, but the first matrix element disagrees.

Using the matrix elements of the prefactor calculated in this paper, it is easy to comment on more general matrix elements which have not yet been studied in the gauge theory. Already in section 2 we mentioned that the tensor structure of the prefactor is such that we expect the matrix elements to change sign when ϕ and ψ impurities are all changed to $D_\mu Z$'s, while we expect vanishing matrix elements when one string has only ϕ or ψ impurities and the other has only $D_\mu Z$ impurities. Another class of amplitudes which were not studied in [7] are those in which the total number of impurities is not conserved in the interaction. From the string theory point of view these interactions are not qualitatively different from the ones in which the number of impurities is conserved, and the matrix elements are easily calculated using the formulas from this paper. For large μ they typically scale as

$$\begin{aligned} 2 \text{ impurities} + 0 \text{ impurities} \to 2 \text{ impurities} &= \mathcal{O}(\mu) \,, \\ 1 \text{ impurity} + 1 \text{ impurity} \to 2 \text{ impurities} &= \mathcal{O}(\mu) \,, \\ 2 \text{ impurities} + 0 \text{ impurities} \to 0 \text{ impurities} &= \mathcal{O}(\mu^2) , \\ 0 \text{ impurities} + 0 \text{ impurities} \to 2 \text{ impurities} &= \mathcal{O}(\mu^2) . \end{aligned} \tag{4.5}$$

Finally, a class of matrix elements which has not yet been studied on either side of the correspondence are those which involve non-primary string states (i.e., states with fermionic oscillators). On the string theory side, one would need to know the matrix elements of the fermionic operator Λ.

Note added. The original version of this paper reported agreement between the matrix elements (4.4) calculated in string field theory and those predicted by the proposal (1.2). The source of the spurious agreement stemmed from a missing i in the quantity $s(p)$ introduced in (3.13) (this has also been pointed out in [18]), such that the i was missing from (3.15). As a consequence, the $F^-_{m(1)} F^-_{n(3)}$ term in (4.2) was reported with the wrong sign. As mentioned in the introduction, the resulting apparent disagreement between light-cone string field theory and the proposal (1.2) in no way disparages the BMN correspondence, but merely indicates that (as mentioned under (2.4)) one should take into account the fact that at finite g_2, the dictionary between single- (double-, etc.) string states and single- (double-, etc.) trace BMN operators gets corrected [19, 20, 21].

We refer the reader to [22] for a clarification of the basis transformation between the gauge theory and string field theory. At the end of the day, it turns out that the prefactor for the cubic hamiltonian H_3 (for string states which are primary in the notation of subsection 2.2) is proportional to

$$\sum_{r=1}^{3}\sum_{n=-\infty}^{\infty}\frac{\omega_{n(r)}}{\alpha_{(r)}}\alpha^{I\dagger}_{n(r)}\alpha^{J}_{-n(r)}v_{IJ}\,, \tag{4.6}$$

where α_n are the oscillators in the BMN basis (see footnote 2) and $v_{IJ} = \mathrm{diag}(1_4, -1_4)$. This simple formula is valid for all μ.

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A. A compendium of matrices and their properties

In this appendix we write down the infinite matrices which appear in the light-cone string vertex and list some of their properties. Some of the matrices are labelled by indices running from $-\infty$ to ∞, while others are labelled by indices running only from 1 to ∞.

We define $\beta = \alpha_{(1)}/\alpha_{(3)}$. The delta-functional overlap involves the matrices [23]

$$\begin{aligned}
A^{(1)}_{mn} &= (-1)^{m+n+1}\frac{2\sqrt{mn}}{\pi}\frac{\beta\sin m\pi\beta}{n^2 - m^2\beta^2}\,,\\
A^{(2)}_{mn} &= (-1)^{m+1}\frac{2\sqrt{mn}}{\pi}\frac{(\beta+1)\sin m\pi\beta}{n^2 - m^2(\beta+1)^2}\,,\\
A^{(3)}_{mn} &= \delta_{mn}\,,\\
C_{mn} &= m\delta_{mn}\,,
\end{aligned} \tag{A.1}$$

and the vector

$$B_m = (-1)^{m+1}\frac{2}{\pi}\frac{\alpha_{(3)}}{\alpha_{(1)}\alpha_{(2)}}m^{-3/2}\sin m\pi\beta\,. \tag{A.2}$$

These are all defined for $m, n > 0$ only. In [23] it was shown that the identities

$$A^{(r)\mathrm{T}}C^{-1}A^{(s)} = -\frac{\alpha_{(r)}}{\alpha_{(3)}}\delta^{rs}C^{-1}\,, \qquad A^{(r)\mathrm{T}}CA^{(s)} = -\frac{\alpha_{(3)}}{\alpha_{(r)}}\delta^{rs}C\,, \qquad A^{(r)\mathrm{T}}CB = 0 \tag{A.3}$$

hold for $r, s \in \{1, 2\}$, and that

$$\begin{aligned}
\sum_{r=1}^{3}\frac{1}{\alpha_{(r)}}A^{(r)}CA^{(r)\mathrm{T}} &= 0\,,\\
\sum_{r=1}^{3}\alpha_{(r)}A^{(r)}C^{-1}A^{(r)\mathrm{T}} &= \frac{1}{2}\alpha_{(1)}\alpha_{(2)}\alpha_{(3)}BB^{\mathrm{T}}\,.
\end{aligned} \tag{A.4}$$

Occasionally we will use the shorthand

$$A^{(r)}_{-} \equiv \frac{\alpha_{(3)}}{\alpha_{(r)}}C^{-1}A^{(r)}C, \tag{A.5}$$

so that the first identity in (A.3) takes the form

$$A_-^{(r)\mathrm{T}} A^{(s)} = -\delta_{rs}\mathbf{1}\,, \qquad r,s \in \{1,2\}\,. \tag{A.6}$$

In [8] these matrices were assembled into the larger matrices $X^{(r)}$, defined for $r = 3$ by $X^{(3)} = \mathbf{1}$ and for $r,s \in \{1,2\}$ by

$$\begin{aligned} X^{(r)}_{mn} &= (C^{1/2}A^{(r)}C^{-1/2})_{mn} \qquad \text{if} m,n>0\,, \\ &= \frac{\alpha_{(3)}}{\alpha_{(r)}}(C^{-1/2}A^{(r)}C^{1/2})_{-m,-n}\,, \qquad \text{if} m,n<0\,, \\ &= -\frac{1}{\sqrt{2}}\epsilon^{rs}\alpha_{(s)}(C^{1/2}B)_m \qquad \text{if} n=0 \text{and } m>0\,, \\ &= 1 \qquad \text{if } m=n=0\,, \\ &= 0 \qquad \text{otherwise}\,. \end{aligned} \tag{A.7}$$

From (A.3) it follows that

$$X^{(r)\mathrm{T}}X^{(s)} = -\frac{\alpha_{(3)}}{\alpha_{(r)}}\delta_{rs}\mathbf{1} \tag{A.8}$$

for $r,s \in \{1,2\}$, while (A.4) implies that

$$\sum_{r=1}^{3}\alpha_{(r)}X^{(r)}X^{(r)\mathrm{T}} = 0\,. \tag{A.9}$$

Next we define the diagonal matrices

$$\left[C_{(r)}\right]_{mn} = \delta_{mn}\omega_{m(r)} = \delta_{mn}\sqrt{m^2+\mu^2\alpha_{(r)}^2}\,. \tag{A.10}$$

We will use these matrices both in formulas where m ranges over all integers as well as in formulas where m ranges only over positive integers—in each formula it should be clear what the intended summation range is. For example, when we define

$$U_{(r)} = C^{-1}(C_{(r)} - \mu\alpha_{(r)}\mathbf{1})\,, \tag{A.11}$$

the presence of C^{-1} makes it clear that this is for positive indices only.

Finally, when investigating the large μ limit of the prefactor, we will need to know

$$\begin{aligned} (A^{(1)\mathrm{T}}C^{-1}B)_n &= 2(-1)^n n^{-3/2}\frac{\beta}{\alpha_{(3)}}\,, \\ (A^{(2)\mathrm{T}}C^{-1}B)_n &= 2n^{-3/2}\frac{\beta+1}{\alpha_{(3)}}\,. \end{aligned} \tag{A.12}$$

These results follow easily from the definitions (A.1) and (A.2) with the help of the sum

$$\sum_{p=1}^{\infty}\frac{1}{p^2}\frac{\sin^2(p\pi\beta)}{n^2-p^2\beta^2} = -\frac{1}{2}\frac{\pi^2}{n^2}\beta(\beta+1)\,, \qquad -1<\beta<0\,. \tag{A.13}$$

B. On the Neumann matrices

The Neumann matrix elements $\overline{N}^{(rs)}_{mn}$ in the pp-wave background were determined in [8]. Formally, they are given by

$$\overline{N}^{(rs)} = \delta^{rs}\mathbf{1} - 2C_{(r)}^{1/2}X^{(r)\mathrm{T}}\Gamma_a^{-1}X^{(s)}C_{(s)}^{1/2}\,, \tag{B.1}$$

where

$$\Gamma_a = \sum_{r=1}^{3} X^{(r)}C_{(r)}X^{(r)\mathrm{T}}\,. \tag{B.2}$$

The formula (B.1) is not convenient for actually calculating matrix elements because the matrix (B.2) does not exist. This technical nuisance arises already in flat space, and can be cured with the help of a trick which we now explain.

From the structure of the X matrices (A.7) it is clear that Γ_a is block diagonal. Using the identities (A.4), it is easy to see that the three blocks can be written as

$$[\Gamma_a]_{mn} = \begin{cases} (C^{1/2}\Gamma_+C^{1/2})_{mn} & m,n>0\,, \\ -2\mu\alpha_{(3)} & m=n=0\,, \\ (C^{1/2}\Gamma_-C^{1/2})_{-m,-n} & m,n<0\,, \end{cases} \tag{B.3}$$

where we have defined

$$\begin{aligned} \Gamma_+ &= \sum_{r=1}^{3} A^{(r)}U_{(r)}A^{(r)\mathrm{T}}\,, \\ \Gamma_- &= \sum_{r=1}^{3} A_-^{(r)}U_{(r)}^{-1}A_-^{(r)\mathrm{T}}\,, \end{aligned} \tag{B.4}$$

and the factors of $C^{1/2}$ are included for convenience in (B.3) so that Γ_+ reduces to the matrix Γ of [23] when μ is set to zero. The matrix Γ_+ exists and is invertible, but the matrix Γ_- does not exist because the matrix product in (B.4) leads to a divergent sum. Fortunately, it is Γ_-^{-1}, not Γ_-, which appears in the Neumann matrices. In order to define Γ_-^{-1}, one can use the identities (A.3) to prove (formally) that

$$\Gamma_+\Gamma_- = \Gamma_+U_{(3)}^{-1} + U_{(3)}\Gamma_-\,. \tag{B.5}$$

Multiplying by Γ_+^{-1} from the left and Γ_-^{-1} from the right leads to an identity which allows us to **define** Γ_-^{-1}:

$$\Gamma_-^{-1} = U_{(3)} - U_{(3)}\Gamma_+^{-1}U_{(3)}\,. \tag{B.6}$$

This matrix Γ_-^{-1} is well-defined but not invertible, despite the unfortunately misleading notation.

Using the definition (B.4) and the identities (A.3), it is easy to show that

$$\Gamma_+^{-1}C^{-1}U_{(3)}A^{(r)} = C^{-1}A_{(r)} + \frac{\alpha_{(r)}}{\alpha_{(3)}}\Gamma_+^{-1}A_{(r)}C^{-1}U_{(3)}\,, \qquad r\in\{1,2\}\,. \tag{B.7}$$

Then, using the definition (B.6), it follows that

$$\Gamma_-^{-1} A_-^{(r)} = -U_{(3)} \Gamma_+^{-1} A_{(r)} U_{(r)} \,, \qquad r \in \{1,2\} \,. \tag{B.8}$$

This remarkable identity allows us to write down simple relations between Neumann matrix elements for positive m, n indices and negative m, n indices. For example, for $r \in \{1,2\}$ and $m, n > 0$ we have

$$\left[\overline{N}^{(r3)}\right]_{-m,-n} = -\left[U_{(r)} \overline{N}^{(r3)} U_{(3)}\right]_{mn} \,, \qquad r \in \{1,2\} \,. \tag{B.9}$$

Relations for other r, s are obtained similarly.

C. On the large μ expansion of the matrix Γ_+

In this appendix we summarize what is known about the behavior of the matrix Γ_a for large μ. This question has been studied in [9, 10]. We demonstrate that matrix elements of the light-cone hamiltonian will typically have terms of order $(\lambda')^{3/2}$, which are puzzling from the field theory point of view.

The matrix Γ_a (B.2) encodes much of the structure of the cubic string vertex, but it is a very complicated function of μ. The large μ expansion is difficult to study because the process of taking $\mu \to \infty$ limit does not commute with the infinite sums in the matrix multiplication which defines Γ_a. A prototype of this phenomenon is the sum

$$\sum_{p=1}^{\infty} \frac{1}{p^2} \frac{1}{1+\lambda p^2} = \frac{\pi^2}{6} - \frac{\pi}{2}\sqrt{\lambda} + \mathcal{O}(\lambda) \,, \tag{C.1}$$

where we have expanded the known result for small λ. Naively expanding the summand would lead one to the incorrect conclusion that only integer powers of λ appear. Of course in this toy example we know how to do the sum (C.1) explicitly. For Γ_a the sums are much more complicated, but we will be able to show that a similar phenomenon occurs.

We start by recalling that

$$\begin{aligned} \Gamma_+ &= \sum_{r=1}^{3} A^{(r)} U_{(r)} A^{(r)\mathrm{T}} \\ &= U_{(3)} + \sum_{r=1}^{2} A^{(r)} U_{(r)} A^{(r)\mathrm{T}}. \end{aligned} \tag{C.2}$$

If we split off the leading term for large μ, which is allowed since it gives a convergent sum, we obtain

$$\begin{aligned} \Gamma_+ &= U_{(3)} + \frac{1}{2\mu} \sum_{r=1}^{2} \frac{1}{\alpha_{(r)}} A^{(r)} C A^{(r)\mathrm{T}} + \sum_{r=1}^{2} A^{(r)} \left(U_{(r)} - \frac{C}{2\mu\alpha_{(r)}} \right) A^{(r)\mathrm{T}} \\ &= U_{(3)} - \frac{1}{2\mu\alpha_{(3)}} C + R \,, \end{aligned} \tag{C.3}$$

where we have used (A.4) to perform one of the summations, and defined the 'remainder' matrix R which has the matrix elements

$$\begin{aligned} R_{mn} &= \sum_{r=1}^{2} \left[A^{(r)} \left(U_{(r)} - \frac{C}{2\mu\alpha_{(r)}} \right) A^{(r)\mathrm{T}} \right]_{mn} \\ &= \frac{4}{\pi^2} (-1)^{m+n} \sqrt{mn} \sin(m\pi\beta) \sin(n\pi\beta) \left(\Sigma_{mn}(\beta) + \Sigma_{mn}(-1-\beta) \right), \end{aligned} \tag{C.4}$$

written in terms of the sum

$$\Sigma_{mn}(\beta) = \beta^2 \sum_{p=1}^{\infty} \frac{p}{(p^2 - m^2\beta^2)(p^2 - n^2\beta^2)} \left[\frac{\sqrt{p^2 + \mu^2 \alpha_{(3)}^2 \beta^2} - \mu\alpha_{(3)}\beta}{p} - \frac{p}{2\mu\alpha_{(3)}\beta} \right]. \tag{C.5}$$

Naively taking μ to infinity before performing the sum would give μ^{-3} times a linearly divergent sum. Instead, it is clear that one must perform the sum first at finite μ (where it is clearly convergent, since the term in brackets behaves like p for large p) and then take the large μ limit of the result. It is easy to estimate that the resulting behavior is of order μ^{-2}, and that there is no cancellation between the two terms in (C.4), so that the matrix R has a nonzero leading term of order μ^{-2}.

Going back to (C.3), we are able to conclude that for large μ,

$$\Gamma_+ = -2\mu\alpha_{(3)} C^{-1} - \frac{1}{\alpha_{(3)}\mu} C + \mathcal{O}\left(\mu^{-2}\right), \tag{C.6}$$

where we have proven that the μ^{-2} term is nonzero, but we have not calculated it explicitly. The authors of [10] used an analytic continuation argument to sum all of the odd powers of μ on the right-hand side of (C.6), obtaining the remarkably simple result

$$\Gamma_+ = 2C_{(3)} C^{-1}. \tag{C.7}$$

Additional terms must be present since it is manifest from (C.2) that Γ_+ goes over smoothly to the flat space matrix Γ from [17] (which has a complicated structure) as $\mu \to 0$. Certainly, it would be very desirable to have better analytic control over the matrix Γ_+ (or even better, Γ_+^{-1}).

The presence of the μ^{-2} term in (C.6) implies the existence of $(\lambda')^{3/2}$ terms in typical light-cone hamiltonian matrix elements, which seem difficult to explain from the gauge theory. Perhaps the prefactor contains half-integer powers of λ' in such a way that these strange powers cancel, or perhaps the dictionary between the string and gauge theories must be modified in an appropriate way at higher order to achieve agreement.

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